Journal of Statistical Physics, Vol. 92, Nos. 5/6, 1998

Book Review: *The Critical Point. A Historical Introduction to the Modern Theory of Critical Phenomena*

The Critical Point. A Historical Introduction to the Modern Theory of Critical Phenomena. Cyril Domb, Taylor and Francis, London, 1996.

In his 1869 Bakerian Lecture to the Royal Society of London, entitled "On the Continuity of the Gaseous and Liquid States of Matter," Thomas Andrews, professor of chemistry at Queen's College, Belfast, reported his study of the pressure-density isotherms of carbon dioxide, with emphasis on their behavior near the point at 31°C and 73 atm at which the liquid and gaseous phases lose their separate identities. Ch. Cagniard de la Tour, almost a half century earlier, had seen that when a sealed tube, initially about a third full of water, alcohol, or ether, was heated, the liquid in it expanded and ultimately "disappeared" into the vapor. That marked the discovery of the critical point; but it was Andrews who recognized this point to be not that of the unsymmetrical disappearance of one of the two phases but rather of their joint merging into one undifferentiated fluid. (For this outline of the history I have called on the account by J. S. Rowlinson in *Nature* **224**:541 (1969), as did the author of this book.) The title of Andrews' lecture was echoed, a few years later, by van der Waals in the title he gave to his then soon-to-be famous doctoral thesis of 1873.

Independently, the centuries-old knowledge that magnets lose their power of attraction when heated to a high enough temperature was made the object of quantitative study and measurement by Pierre Curie, near the end of the nineteenth century. This became to the science of magnetism

---

This review first appeared in a French translation by P. G. de Gennes in the *Bulletin de la Société Française de Physique* **112**:50 (1998).

0022-4715/98/0900-1213$15.00/0

what Andrews' researches were to that of fluids. It is the story of the confluence of these two streams and of the remarkable advances in the science of condensed matter to which it led that Cyril Domb tells in his wonderful book "The Critical Point."

A huge leap forward in the theory of such phenomena came with what Domb calls the "Onsager revolution," which was Onsager's publication in 1944 of his calculation of the partition function of the two-dimensional Ising model, a lattice spin model of a ferromagnet (or a corresponding model of liquid-vapor equilibrium). Not long after, starting with his Ph.D. thesis in 1949, Domb, with characteristic courage and perseverance, undertook the systematic derivation of high- and low-temperature series expansions of unprecedented length for the thermodynamic functions of such lattice models in three dimensions, correlating the coefficients in the expansions with the models' critical-point properties. This pioneering program gave us the first glimpses of what the three-dimensional models were doing and showed that they were capable of accounting for the persistent–and until then mysterious–discrepancies between the classical mean-field theories and experiment. Pursuit of the theory behind those discrepancies was to become a major industry in academic science for decades; Domb was "present at the Creation." Notable among the many supremely capable and productive members of the school of research Domb founded was M. E. Fisher, who has himself gone on to make contributions of the most profound importance to the field and who has written an eloquent foreword to this book.

"The Critical Point" had its origins in a course of lectures Professor Domb has given at London University and at Bar-Ilan University in Israel, where he is now. It may be viewed also as a greatly expanded version of his article "Critical Phenomena" in the Encyclopedia of Applied Physics. It is a lively and authoritative account of the history and present state of our understanding of equilibrium critical-point phenomena. The book starts with an historical survey going back to the very beginnings of statistical mechanics. The author has always had a deep interest in the history of science–he has elsewhere written extensively on Maxwell and the physics of that time–and the present work is very thorough on the history (as promised by the book's subtitle) as well as on the objective scientific content of the subject. Here among his historical remarks are even a few captivating personal touches, such as a recounting of a conversation he had with Bragg on the origins of the Bragg–Williams theory and of an exchange of correspondence with Onsager about some work of Fisher's. He puts right some misconceptions about the history, such as the present use of "Curie point," which unfairly overlooks Hopkinson's earlier recognition of the "critical point" of a ferromagnet (although Domb acknowledges and documents extensively the real achievements of the great French physicist in this field). He recalls for us the heroic 1960s, when many exciting advances in the subject of critical phenomena were taking place, and when elements of our understanding that are now taken for granted were then just becoming clear or were still being debated.

The historical survey with which the book begins is followed by chapters on the "classical" (largely mean-field) theories of the phase transitions and critical points of fluids and of magnets, and of fluctuations, correlations, and light scattering (including x-ray and neutron scattering in solids). These chapters are then followed by the one on the "Onsager revolution," and that, in turn, by "Reconciliation"–essentially the scaling and homogeneity principles, including the "scaling laws" that connect the critical-point exponents to each other. The final full chapter is on the renormalization-group theory of Wilson (including the Wilson-Fisher ε-expansion), treated as the culmination of the theory's evolution. Every topic has been carefully thought and worked through by the author and expounded with all its technical detail. The book is replete with references. In the few cases where the author follows another's treatment he is punctilious in quoting his sources. An appendix includes brief discussions of a few related topics, including Monte Carlo simulations, percolation theory, tricritical points, and the connection, discovered by de Gennes, between the self-avoiding walk problem of polymer theory and the magnetic models.

The (older!) physical chemists among the book's readers may notice two small omissions from the history. Professor Domb has woven together, both in their historical and in their scientific contexts, the threads of the critical points of pure fluids and of magnets, and of the order-disorder transitions of binary alloys, but he does not refer to the critical consolute points of liquid mixtures. In the late 1940s and early 1950s O. K. Rice and B. H. Zimm remarked that these systems' coexistence curves were characterized by the same critical-point exponent, and hence the same deviation from classical theory, as were those for liquid-vapor equilibrium in one-component fluids, and that with the proper scaling the respective coexistence curves could be made to coincide. In addition, in that period both Rice and R. L. Scott knew that deviations from classical critical-point behavior in the several thermodynamic properties were not independent of each other and they found early (albeit incomplete) versions of the scaling laws connecting the corresponding exponents. But such lacunae are a minor matter in light of the vast treasure of history and of science that is here.

"The Critical Point" by Cyril Domb is going to be and is going long to remain a definitive reference work on the statistical mechanics of critical phenomena. It is scholarship of the highest order.

Benjamin Widom
Department of Chemistry
Cornell University
Baker Laboratory
Ithaca, New York 14853-1301